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Statistics and Operational Research
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The Effect of Recovery Algorithms on Compressive Sensing Background Subtraction.

EPSRC
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Motivation

"Big Brother is Watching You."
- George Orwell, 1984

Background Subtraction

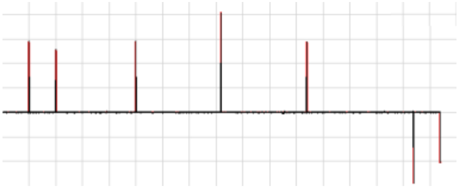
- Construct, update then subtract.
- Not new - many methods exists.
- Most traditional methods are not efficient.
- CCTV often slowly adaptive background + rare foreground (spatially and temporally)
- Waste of resources

What is compressive sensing?

Compressive sensing is a method of **reducing the amount of data collected** from a signal without compromising the ability to later **reconstruct the signal accurately.** This method will only work if the signal of interest is compressible.

Sparse and Compressible Signals

- A signal is known as being **K-sparse** if $\mathbf{x} \in R^N$ can be represented as a linear combination of K basis vectors.
- Interested in $K \ll N$.
- If a signal is **compressible** there exist K large coefficients but the remaining $N - K$ coefficients are only required to be small and not necessarily zero.



The encoding process.

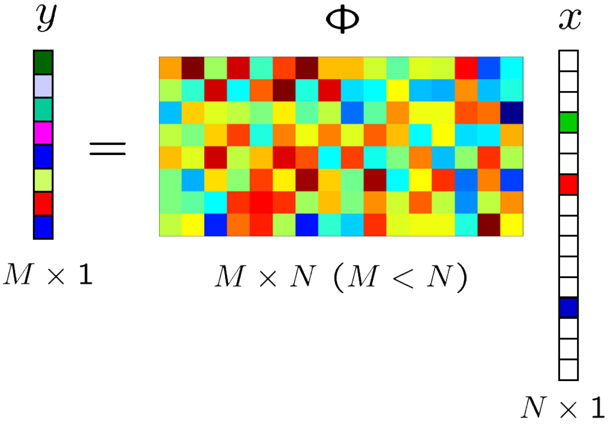


Figure: CS measurement process, courtesy of Volkan Cevher.

Restricted Isometry Property (RIP)

A matrix $\mathbf{\Phi}$ satisfies the (RIP) of order K if there exists a $\delta_K \in (0,1)$ such that

$$(1-\delta_k)\|\mathbf{x}\|_2^2 \leq \|\mathbf{\Phi x}\|_2^2 \leq (1+\delta_k)\|\mathbf{x}\|_2^2,$$

for all $\mathbf{x} \in \sum_K = \{\mathbf{x} : \|\mathbf{x}\|_0 \leq K\}$,

where $\|\mathbf{x}\|_0$ is the zero pseudo-norm defined as

$$\|\mathbf{x}\|_0 = \#(i|x_i \neq 0).$$

If $\mathbf{\Phi}$ satisfies the RIP with order $2K$, then $\mathbf{\Phi}$ approximately preserves the distance between any pair of K-sparse vectors. Unfortunately the task of checking that a matrix satisfies the RIP is a NP-hard problem, but fortunately the RIP will hold true with high probability if $\mathbf{\Phi}$ is selected as a random matrix and $M \geq cK \log \frac{N}{K}$, where c is a small constant.

Recovery of sparse transforms

- Solve $\mathbf{y} = \mathbf{\Phi x}$, infinitely many solutions! Fat $\mathbf{\Phi}$ implies underdetermined system.
- We know that $\mathbf{x}$ was **sparse**
- What algorithms can we use to decode?
- Convex Optimisation or Greedy Algorithms or something else...?

Using ℓ_1 minimization to promote sparsity

$$\|\mathbf{x}\|_1 = \sum_{i=1}^{N} |x_i|$$

Originally used in geophysics to aid detection of sparse spike trends in earthquake data, optimisation based on the l_1 norm can closely approximate compressible signals with high probability.

$$\min_{\mathbf{x}} ||\mathbf{x}||_1 \text{ subject to } \mathbf{y} = \mathbf{\Phi}\mathbf{x}.$$

Orthogonal Matching Pursuit

Algorithm 1 Orthogonal Matching Pursuit

Define the columns of $\mathbf{\Phi}$ to be $\varphi_1, \varphi_2, \ldots, \varphi_N$.

Require: $\mathbf{r_0} = \mathbf{y}, \Lambda_0 = \emptyset$ and iteration counter $i = 1$

for $i < T$ **do**

$\lambda_t = \text{argmax}\ _{j=1,\ldots,N} | < r_{t-1}, \varphi_j > |$

{Find the index for the column of Φ with the greatest contribution.}

$\Lambda_t = \Lambda_{t-1} \cup \lambda_t,\ \Phi_t = [\mathbf{\Phi_{t-1}}, \varphi_{\lambda_t}]$

{Keeps track of the columns used.}

$\mathbf{x_t} = \text{argmin}_{\mathbf{x}} || \mathbf{y} - \mathbf{\Phi_t x} ||_2$

{Updates the signal estimate.}

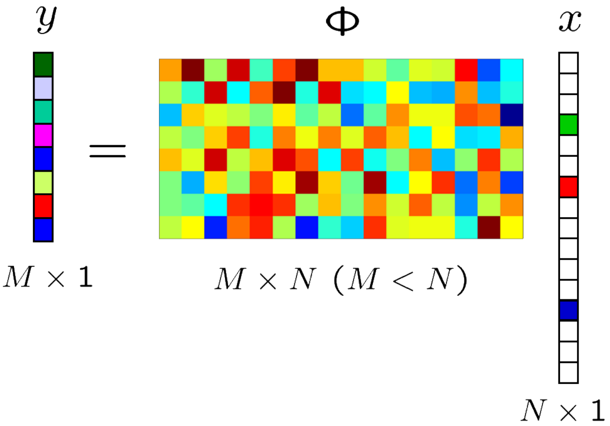
$\mathbf{r_t} = \mathbf{y} - \mathbf{\Phi_t x_t}$

{Updates the measurement residual.}

end for

return $\hat{\mathbf{x}}$

Recap



Sparsity

(a) Test frame (b) Ground truth

Figure: The spatial sparsity of foreground. A frame from the PETS data set and the corresponding foreground in white. In this example, less than 1% of the frame is foreground, as N=442,368 and K=3862.

Encoding

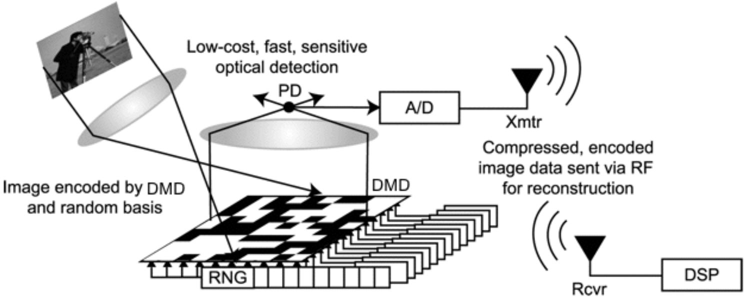
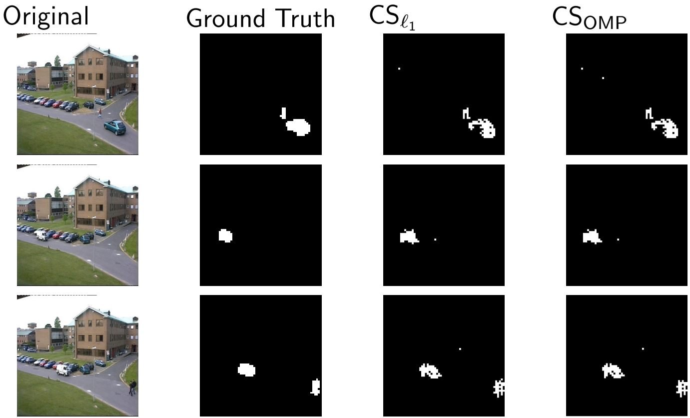


Figure: The single pixel camera

Compressive Sensing Background Subtraction.

Require: Initial compressed background y_0^b
  for all t **do**
    Compressively Sense (Encode) $y_t = \Phi x_t$.
    Reconstruct (Decode) $\hat{x}_t = \Delta(y_t - y_t^b)$
    Update Background $y_{t+1}^b = \alpha y_{t+1} + (1-\alpha) y_t^b$
    return $\hat{\mathbf{x}}_\mathbf{t}$
  end for

Results



Results

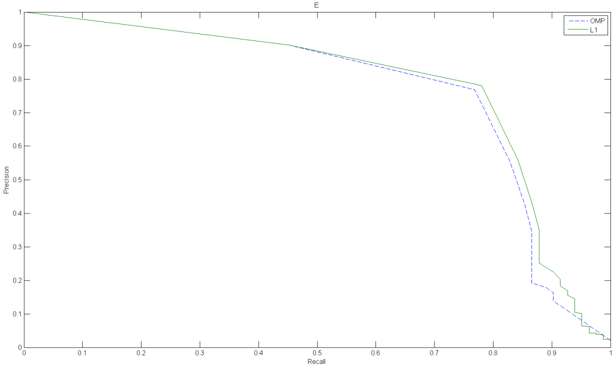


Figure: Precision-Recall Curves for the 3rd test frame

Results

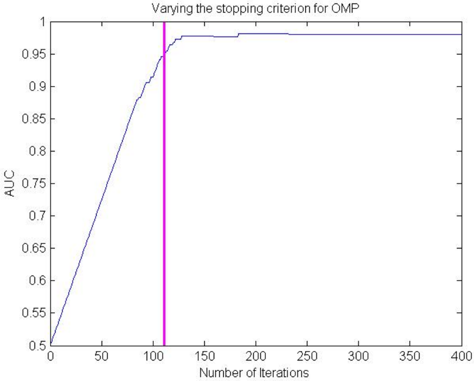


Figure: Selection of the stopping criterion for OMP

Results

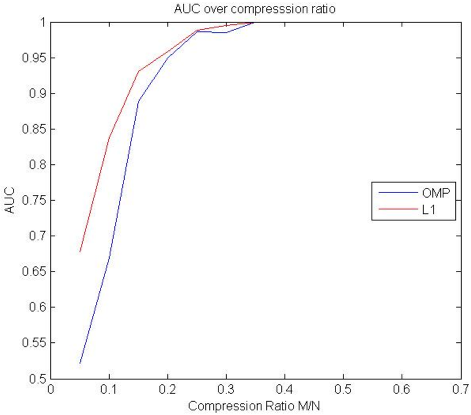


Figure: AUC over Sparsity Ratio

Conclusion and thoughts for the future.

- ℓ_1 minimisation outperforms OMP, but it's close!
- Effect of the stopping criterion is vital for OMP - adaptive methods needed?
- Ideal boundaries for compression ratio of $\frac{M}{N}$ around $25\% - 35\%$
- Can we incorporate prior information to aid the recovery process?

Thanks!

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Precision and Recall Definitions

Recall is defined as the fraction of correctly identified foreground pixels over the number of ground truth foreground pixels which can be written mathematically as

$$\text{Recall} = \frac{TP}{TP + FN}. \tag{1}$$

Precision is defined to be the fraction of correctly identified foreground pixels over the number of detected foreground pixels in total, or when written mathematically

$$\text{Precision} = \frac{TP}{TP + FP}. \tag{2}$$