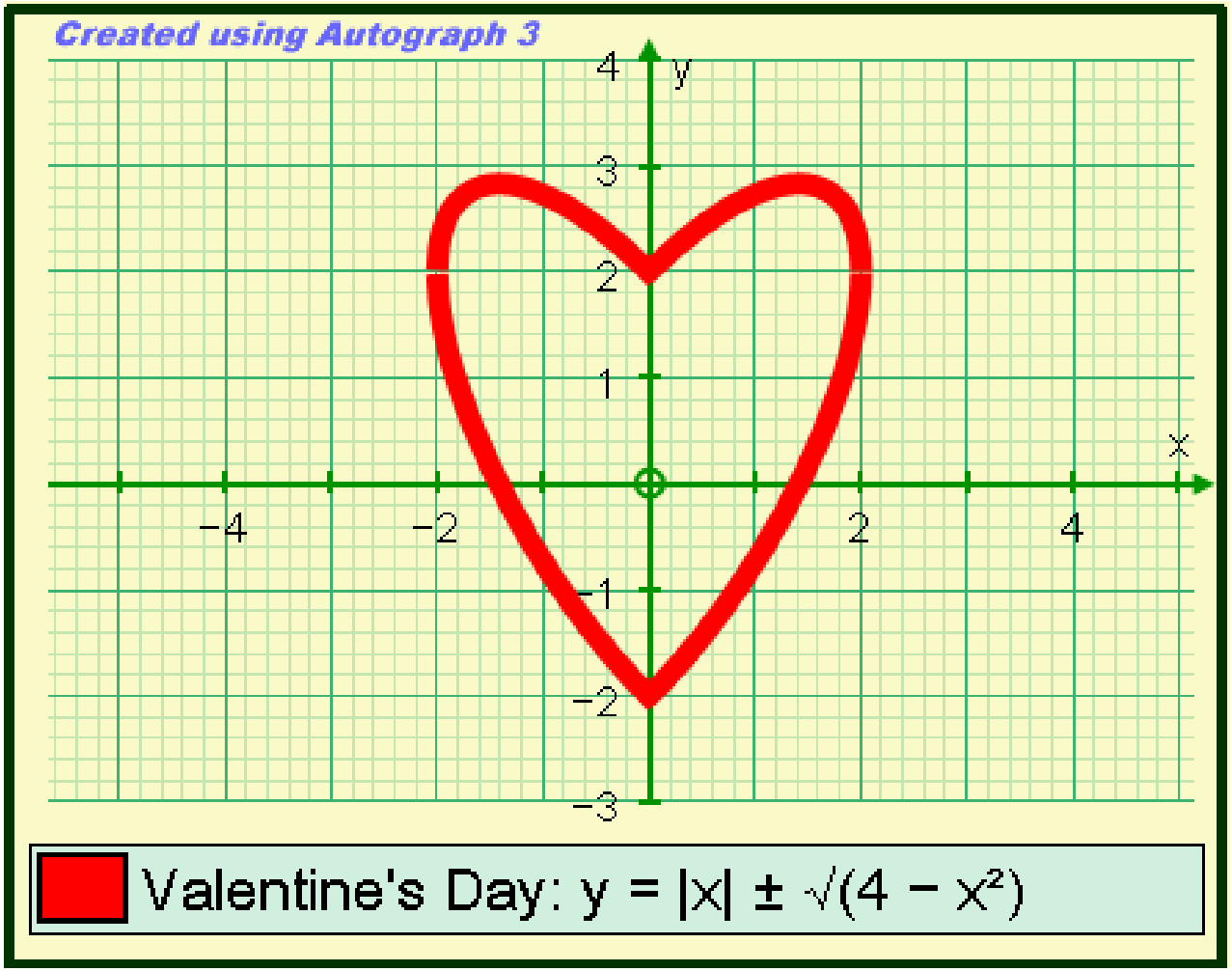
Valentine’s Day maths



$$9x - 7i > 3(3x - 7u)$$

$$i < 3u$$

Mobius Hearts

Amicable Numbers

1
2
4
5
10
11
20
22
44
55
110

1
2
4
71
142
284

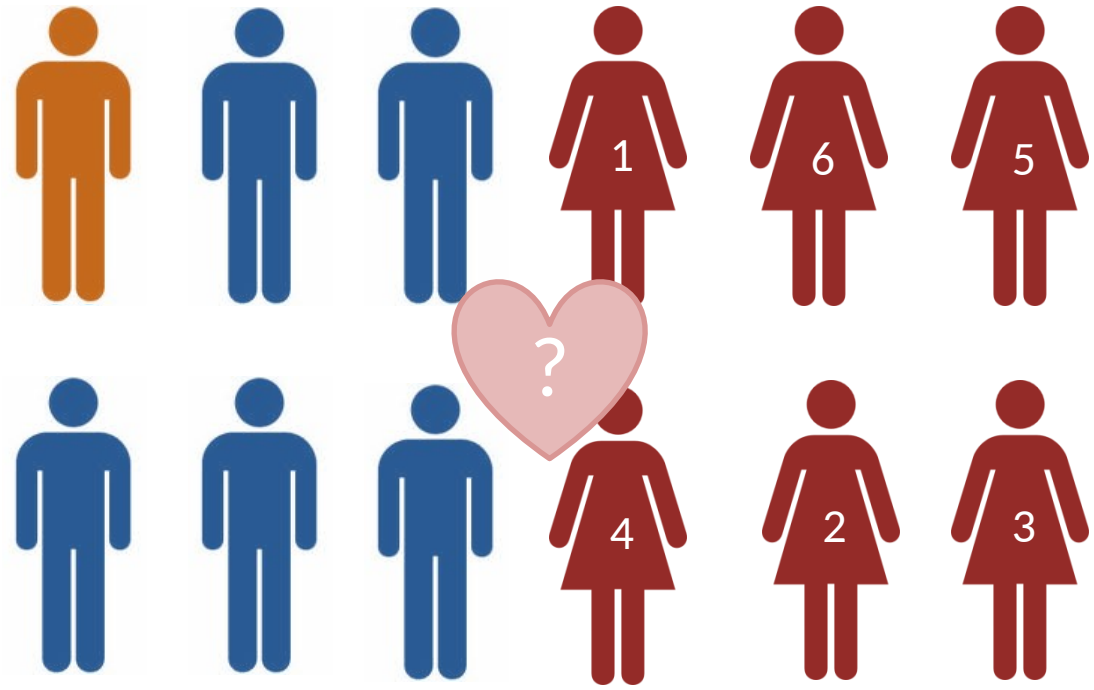
Why Peter Backus can’t get a girlfriend

$$N = R^* \times f_p \times n_e \times f_\ell \times f_i \times f_c \times L$$

- a woman between the age of 24 and 34
- with a university degree
- whom he finds attractive
- she must find him attractive
- she must herself be single
- he must get along with her

The Stable Marriage Problem

Scenario

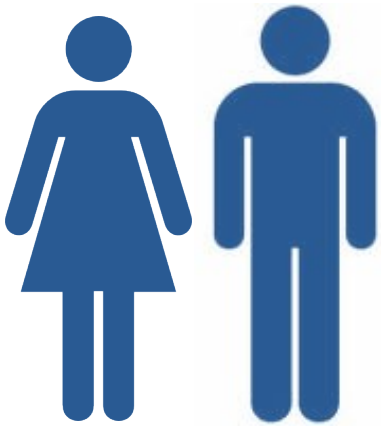


How do we pair them off?

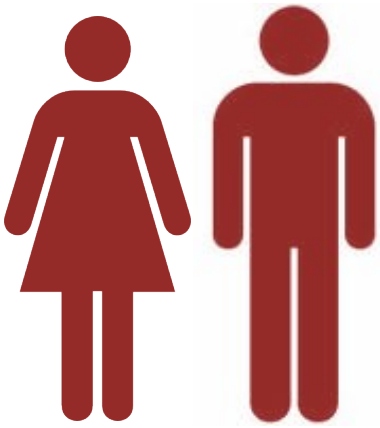
What is a stable marriage situation?

- A matching of men and women such that no two people of opposite sex would **both** rather have each other than their current partners.

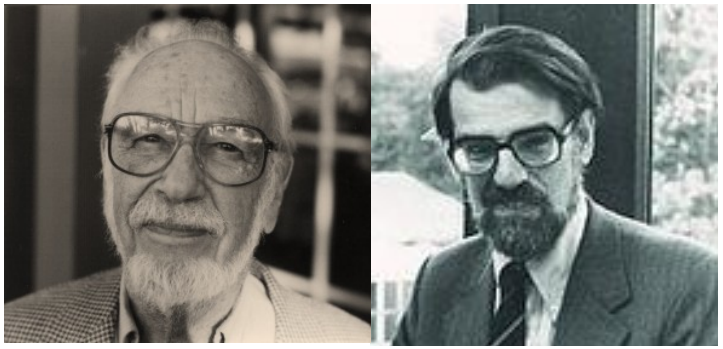
Bobetta Bob



Alice Alan



Gale – Shapley algorithm (1962)



Oberwolfach
Photo Collection

- Male proposes to his preferred female
- If available she **always** accepts his proposal, and we consider the next male.
- If already engaged she chooses her favourite. The rejected male must now propose to his next preference.

A WXYZ

B XWYZ

C WZYX

D ZYWX

W ABCD

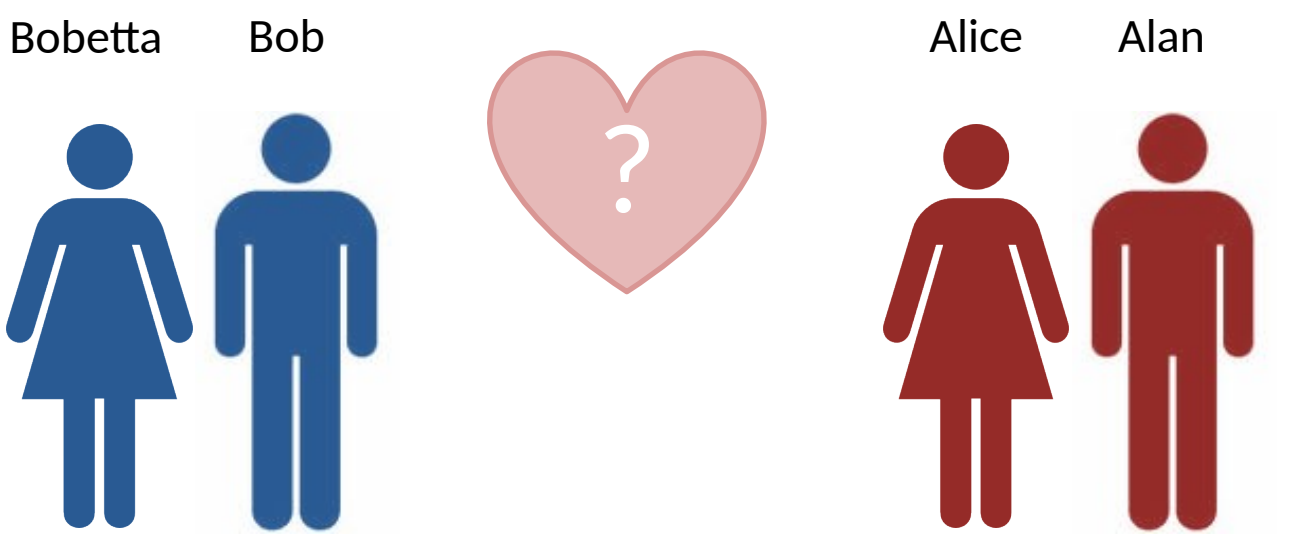
X DCBA

Y ABCD

Z CDAB

	W ABCD	X DCBA	Y ABCD	Z CDAB
A WXYZ				
B XWYZ				
C WZYX				
D ZYWX				

Is it always stable?



Proof that G-S algorithm gives a stable pairing.

- Suppose Bob prefers Alice to his wife Bobetta
- During G-S, Bob must have proposed to Alice before proposing to Bobetta.
- If Alice accepted his proposal, yet is not married to him at the end, she must have dumped him for someone she likes more, and therefore doesn't like Bob more than Alan.
- If Alice rejected his proposal, she was already with someone she liked more than Bob.

	W ABCD	X DCBA	Y ABCD	Z CDAB
A WXYZ				
B XWYZ				
C WZYX				
D ZYWX				


A
W X Y Z
W X Y Z
B
X W Y Z
X W Y Z
C
W Z Y X
W Z Y X
D
Z Y W X
Z Y W X
W
A B C D
A B C D
X
D C B A
D C B A
Y
A B C D
A B C D
Z
C D A B
C D A B

Other applications

- Stable roommates problem
- Medical interns and hospitals

Conclusion

Don’t wait to be asked,
always make the first move!

References

- D. Gale and L. S. Shapley, *College admissions and the stability of marriage*, American Mathematical Monthly 69 (1962), 9-15

- http://en.wikipedia.org/wiki/Stable_marriage_problem